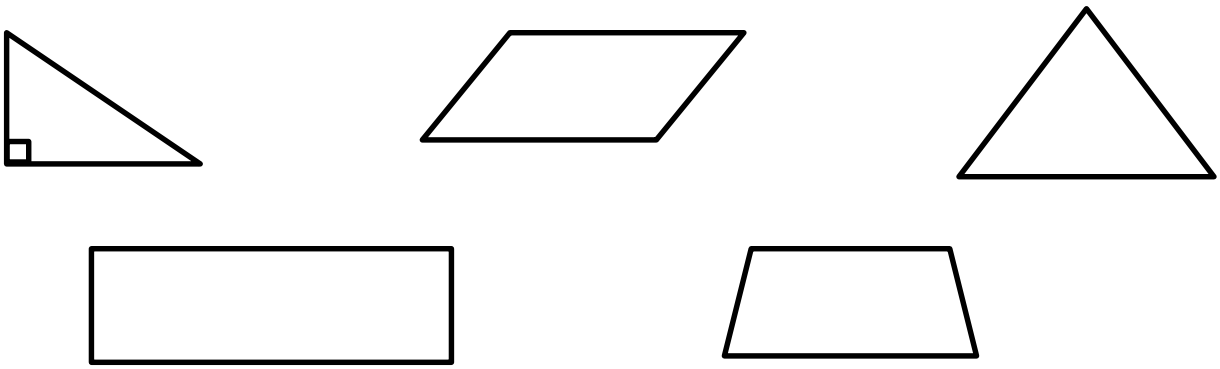
AREA, SURFACE AREA AND VOLUME 6TH GRADE MATH

Lesson 1: The Area of Parallelograms Through Rectangle Facts

Classwork

Opening Exercise

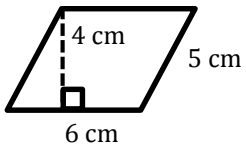
Name each shape.



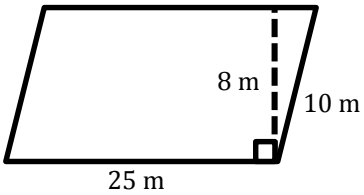
Exercises

1. Find the area of each parallelogram below. Note that the figures are not drawn to scale.

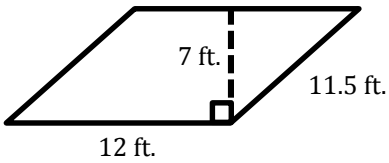
a.



b.



c.



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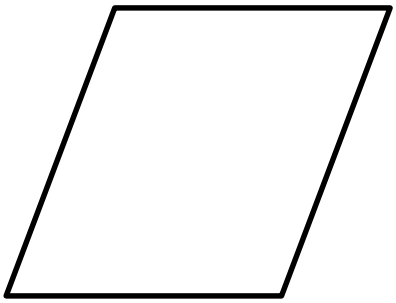
2. Draw and label the height of each parallelogram. Use the correct mathematical tool to measure (in inches) the base and height, and calculate the area of each parallelogram.

a.

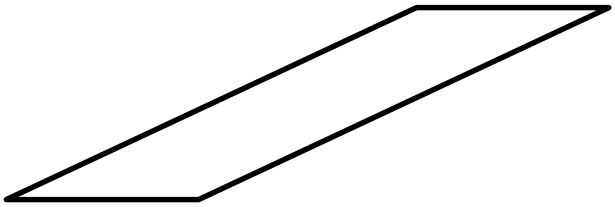
base

b.



base

c.



base

3. If the area of a parallelogram is 42 cm^2 and the height is 7 cm, write an equation that relates the height, base, and area of the parallelogram. Solve the equation.

4. If the area of a parallelogram is $\frac{35}{42} \text{ cm}^2$ and the height is $\frac{1}{7} \text{ cm}$, write an equation that relates the height, base, and area of the parallelogram. Solve the equation.

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Lesson 2: The Area of Right Triangles

Classwork

Exploratory Challenge

a. Use the shapes labeled with an X to predict the formula needed to calculate the area of a right triangle. Explain your prediction.

Formula for the area of right triangles: _____

Area of the given triangle: _____

b. Use the shapes labeled with a Y to determine if the formula you discovered in part (a) is correct.

Does your area formula for triangle Y match the formula you got for triangle X?

If so, do you believe you have the correct formula needed to calculate the area of a right triangle? Why or why not?

If not, which formula do you think is correct? Why?

Area of the given triangle: _____

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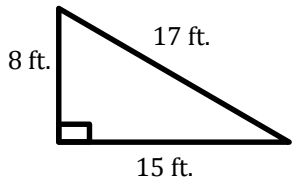
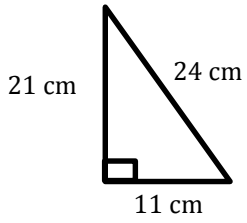
Exercises

Calculate the area of each triangle below. Each figure is not drawn to scale.

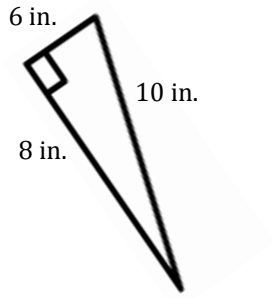
1.



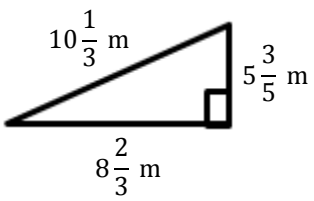
2.



3.



4.



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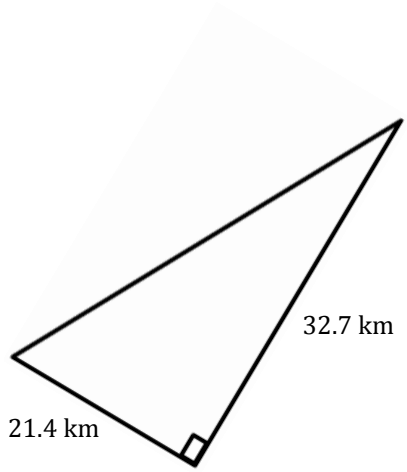
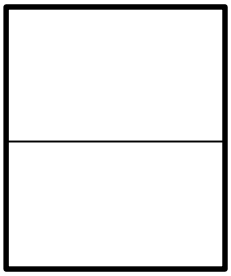
5.

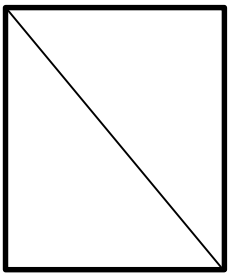


6. Mr. Jones told his students they each need a half of a piece of paper. Calvin cut his piece of paper horizontally and Matthew cut his piece of paper diagonally. Which student has the larger area on their half piece of paper? Explain.

Calvin's Paper



Matthew's Paper



7. Ben requested that the rectangular stage be split into two equal sections for the upcoming school play. The only instruction he gave was that he needed the area of each section to be half of the original size. If Ben wants the stage to be split into two right triangles, did he provide enough information? Why or why not?

8. If the area of a right triangle is 6.22 sq. in. and its base is 3.11 in., write an equation that relates the area to the height, h, and the base. Solve the equation to determine the height.

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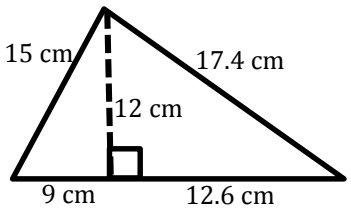
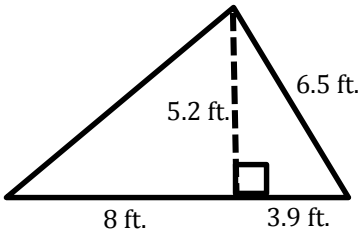
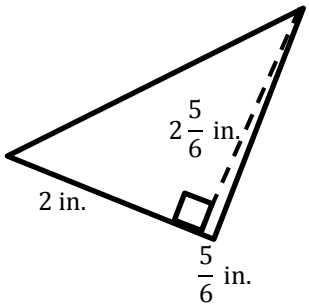
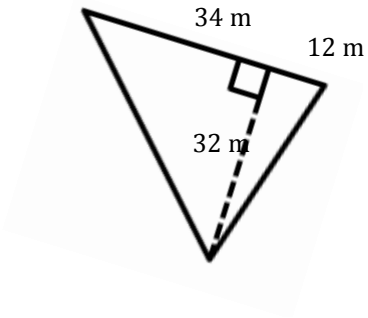
Lesson 3: The Area of Acute Triangles Using Height and Base

Classwork

1. Work with a partner on the exercises below. Determine if the area formula $A = \frac{1}{2}bh$ is always correct. You may use a calculator, but be sure to record your work on your paper as well. Figures are not drawn to scale.

	Area of Two Right Triangles	Area of Entire Triangle
		
		
		
		

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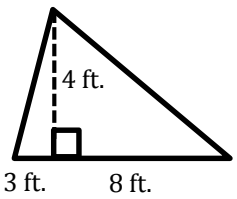
2. Can we use the formula $A = \frac{1}{2} \times \text{base} \times \text{height}$ to calculate the area of triangles that are not right triangles? Explain your thinking.

3. Examine the given triangle and expression.

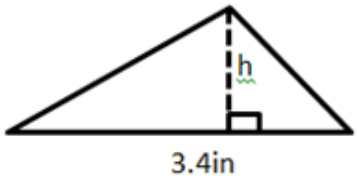
$$\frac{1}{2}(11 \text{ ft.})(4 \text{ ft.})$$



Explain what each part of the expression represents according to the triangle.

4. Joe found the area of a triangle by writing $A = \frac{1}{2}(11 \text{ in.})(4 \text{ in.})$, while Kaitlyn found the area by writing $A = \frac{1}{2}(3 \text{ in.})(4 \text{ in.}) + \frac{1}{2}(8 \text{ in.})(4 \text{ in.})$. Explain how each student approached the problem.

5. The triangle below has an area of 4.76 sq. in. If the base is 3.4 in., let h be the height in inches.



a. Explain how the equation $4.76 \text{ in}^2 = \frac{1}{2}(3.4 \text{ in.})h$ represents the situation.

b. Solve the equation.

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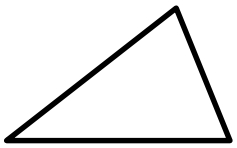
Lesson 4: The Area of All Triangles Using Height and Base

Classwork

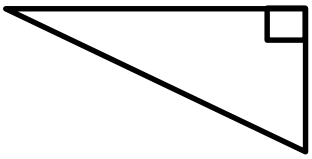
Opening Exercises

Draw and label the altitude of each triangle below.

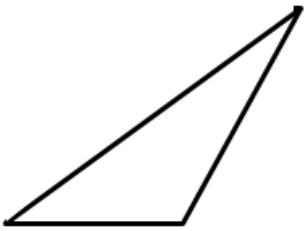
1.



2.



3.



Exploratory Challenge/Exercises 4-8

4. Use rectangle X and the triangle with the altitude inside (triangle X) to show that the area formula for the triangle is $A = \frac{1}{2} \times \text{base} \times \text{height}$.

   a. Step One: Find the area of rectangle X.

   b. Step Two: What is half the area of rectangle X?

   c. Step Three: Prove, by decomposing triangle X, that it is the same as half of rectangle X. Please glue your decomposed triangle onto a separate sheet of paper. Glue it next to rectangle X. What conclusions can you make about the triangle's area compared to the rectangle's area?

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5. Use rectangle Y and the triangle with a side that is the altitude (triangle Y) to show the area formula for the triangle is $A = \frac{1}{2} \times \text{base} \times \text{height}$.
   a. Step One: Find the area of rectangle Y.

   b. Step Two: What is half the area of rectangle Y?

   c. Step Three: Prove, by decomposing triangle Y, that it is the same as half of rectangle Y. Please glue your decomposed triangle onto a separate sheet of paper. Glue it next to rectangle Y. What conclusions can you make about the triangle's area compared to the rectangle's area?

6. Use rectangle Z and the triangle with the altitude outside (triangle Z) to show the area formula for the triangle is $A = \frac{1}{2} \times \text{base} \times \text{height}$.
   a. Step One: Find the area of rectangle Z.

   b. Step Two: What is half the area of rectangle Z?

   c. Step Three: Prove, by decomposing triangle Z, that it is the same as half of rectangle Z. Please glue your decomposed triangle onto a separate sheet of paper. Glue it next to rectangle Z. What conclusions can you make about the triangle's area compared to the rectangle's area?

7. When finding the area of a triangle, does it matter where the altitude is located?

8. How can you determine which part of the triangle is the base and which is the height?

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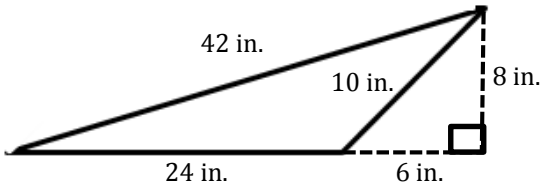
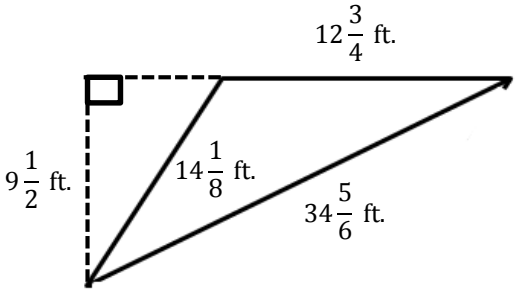
Exercises 9-11

Calculate the area of each triangle. Figures are not drawn to scale.

9.



10.



11. Draw three triangles (acute, right, and obtuse) that have the same area. Explain how you know they have the same area.

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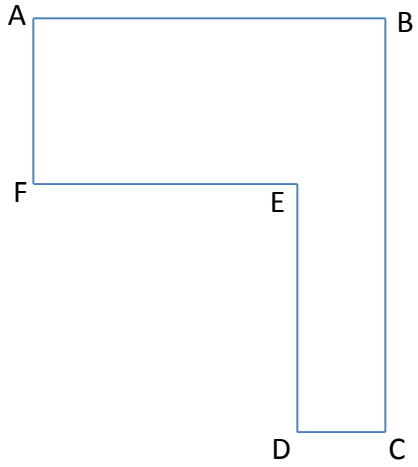
Lesson 5: The Area of Polygons Through Composition and Decomposition

Classwork

Opening Exercise

Here is an aerial view of a woodlot.



If $AB = 10$ units, $FE = 8$ units, $AF = 6$ units, and $DE = 7$ units, find the lengths of both other sides.

$DC =$

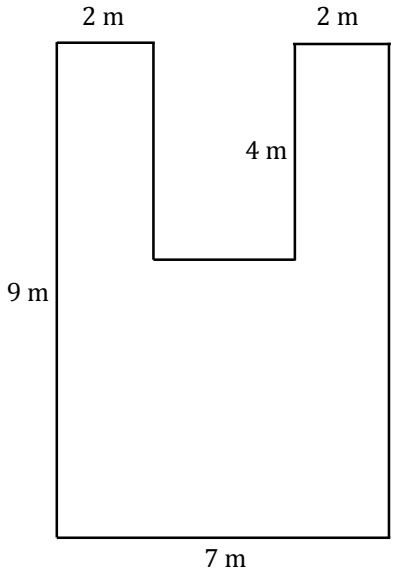
$BC =$

If $DC = 10$ units, $FE = 30$ units, $AF = 28$ units, and $BC = 54$ units, find the lengths of both other sides.

$AB =$

$DE =$

Discussion



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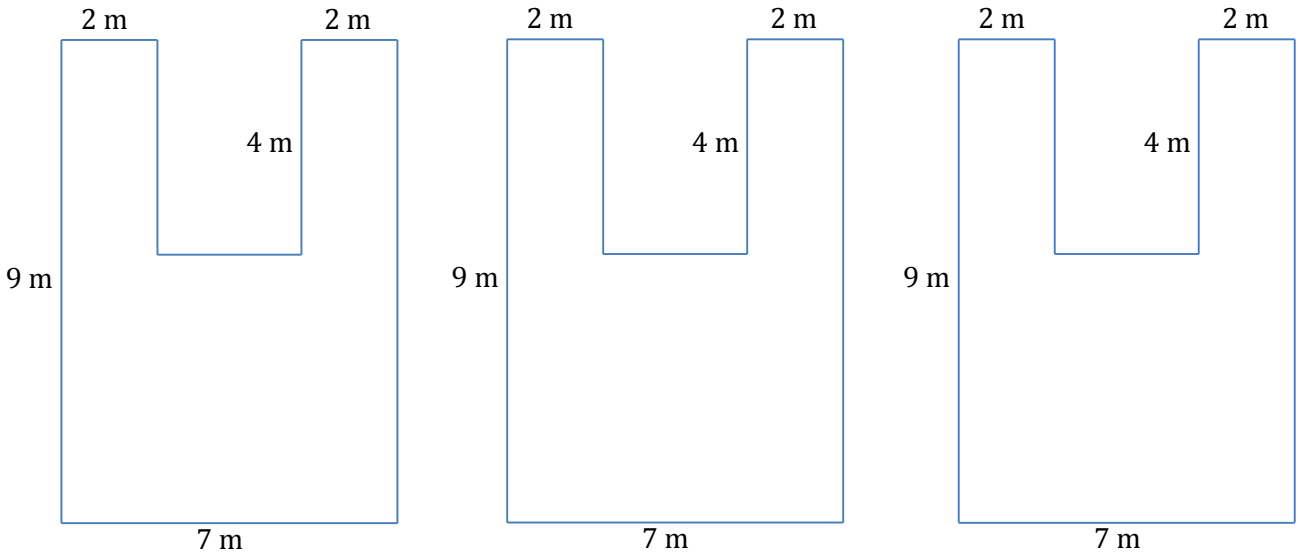
Example 1: Decomposing Polygons into Rectangles

The Intermediate School is producing a play that needs a special stage built. A diagram is shown below (not to scale).

a. On the first diagram, divide the stage into three rectangles using two horizontal lines. Find the dimensions of these rectangles and calculate the area of each. Then, find the total area of the stage.

b. On the second diagram, divide the stage into three rectangles using two vertical lines. Find the dimensions of these rectangles and calculate the area of each. Then, find the total area of the stage.

c. On the third diagram, divide the stage into three rectangles using one horizontal line and one vertical line. Find the dimensions of these rectangles and calculate the area of each. Then, find the total area of the stage.



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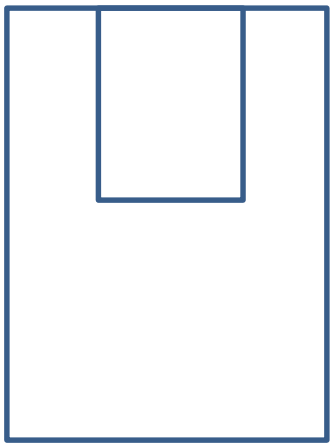
d. Consider this as a large rectangle with a piece removed.



i. What are the dimensions of the large rectangle and the small rectangle?

ii. What are the areas of the two rectangles?

iii. What operation is needed to find the area of the original figure?

iv. What is the difference in area between the two rectangles?

v. What do you notice about your answers to (a), (b), (c), and (d)?

vi. Why do you think this is true?

Example 2: Decomposing Polygons into Rectangles and Triangles

Parallelogram $ABCD$ is part of a large solar power experiment. The base measures 6 m and the height is 4 m.

a. Draw a diagonal from A to C. Find the area of both triangles ABC and ACD.

b. Draw in the other diagonal, from B to D. Find the area of both triangles ABD and BCD.

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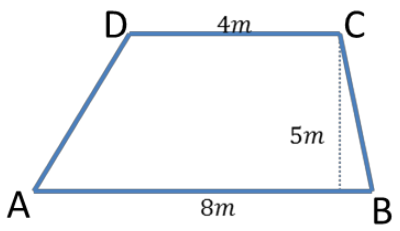
Example 3: Decomposing Trapezoids

The trapezoid below is a scale drawing of a garden plot.



Find the area of both triangles ABC and ACD. Then find the area of the trapezoid.

Find the area of both triangles ABD and BCD. Then find the area of the trapezoid.

How else can we find this area?

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Lesson 6: Area in the Real World

Classwork

Exploratory Challenge 1: Classroom Wall Paint

The custodians are considering painting our classroom next summer. In order to know how much paint they must buy, the custodians need to know the total surface area of the walls. Why do you think they need to know this and how can we find the information?

Make a prediction of how many square feet of painted surface there are on one wall in the room. If the floor has square tiles, these can be used as a guide.

Estimate the dimensions and the area. Predict the area before you measure.

My prediction: _____ ft^2.

a. Measure and sketch one classroom wall. Include measurements of windows, doors, or anything else that would not be painted.

Sketch:

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Object or item to be measured	Measurement units	Precision (measure to the nearest)	Length	Width	Expression that shows the area	Area
door	feet	half foot	$6\frac{1}{2}$ ft.	$3\frac{1}{2}$ ft.	$6\frac{1}{2}$ ft. $\times 3\frac{1}{2}$ ft.	$22\frac{3}{4}$ ft^2

b. Work with your partners and your sketch of the wall to determine the area that will need paint. Show your sketch and calculations below; clearly mark your measurements and area calculations.

c. A gallon of paint covers about 350 ft^2. Write an expression that shows the total area. Evaluate it to find how much paint will be needed to paint the wall.

d. How many gallons of paint would need to be purchased to paint the wall?

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Exploratory Challenge 2

Object or item to be measured	Measurement units	Precision (measure to the nearest)	Length	Width	Area
door	feet	half foot	$6\frac{1}{2}$ ft.	$3\frac{1}{2}$ ft.	$22\frac{3}{4}$ ft^2

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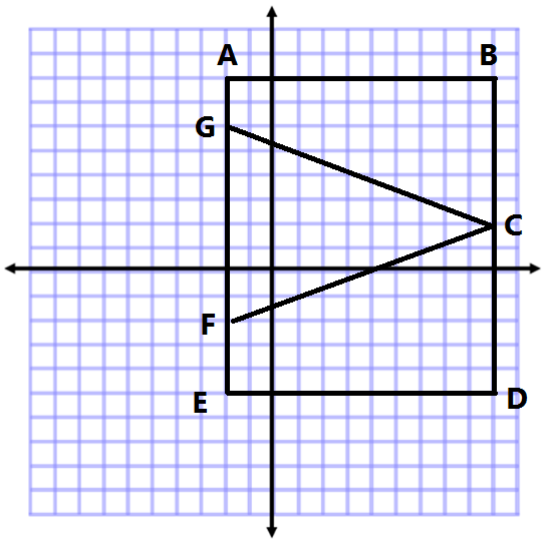
Lesson 7: Distance on the Coordinate Plane

Classwork

Example

Determine the lengths of the given line segments by determining the distance between the two endpoints.

Line Segment	Point	Point	Distance	Proof
$\overline{AB}$				
$\overline{BC}$				
$\overline{CD}$				
$\overline{BD}$				
$\overline{DE}$				
$\overline{EF}$				
$\overline{FG}$				
$\overline{EG}$				
$\overline{GA}$				
$\overline{FA}$				
$\overline{EA}$				



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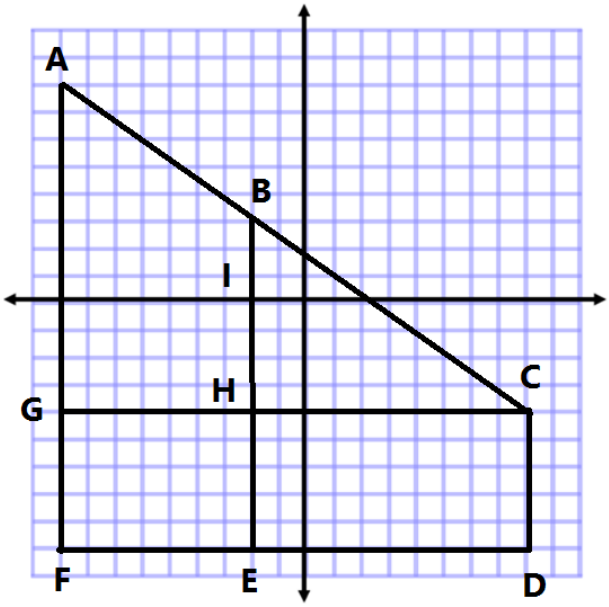
Exercise

Complete the table using the diagram on the coordinate plane.



Line Segment	Point	Point	Distance	Proof
$\overline{BI}$				
$\overline{BH}$				
$\overline{BE}$				
$\overline{GH}$				
$\overline{HC}$				
$\overline{GC}$				
$\overline{CD}$				
$\overline{FG}$				
$\overline{GA}$				
$\overline{AF}$				

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Extension

For each problem below, write the coordinates of two points that are 5 units apart with the segment connecting these points having the following characteristics.

a. The segment is vertical.

b. The segment intersects the x-axis.

c. The segment intersects the y-axis.

d. The segment is vertical and lies above the x-axis.

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Lesson 8: Drawing Polygons on the Coordinate Plane

Classwork

Examples

1. Plot and connect the points $A(3,2)$, $B(3,7)$, and $C(8,2)$. Name the shape, and determine the area of the polygon.

2. Plot and connect the points $E(-8,8)$, $F(-2,5)$, and $G(-7,2)$. Then give the best name for the polygon, and determine the area.

3. Plot and connect the following points: $K(-9,-7)$, $L(-4,-2)$, $M(-1,-5)$, and $N(-5,-5)$. Give the best name for the polygon, and determine the area.

4. Plot and connect the following points: $P(1,-4)$, $Q(5,-2)$, $R(9,-4)$, $S(7,-8)$, and $T(3,-8)$. Give the best name for the polygon, and determine the area.

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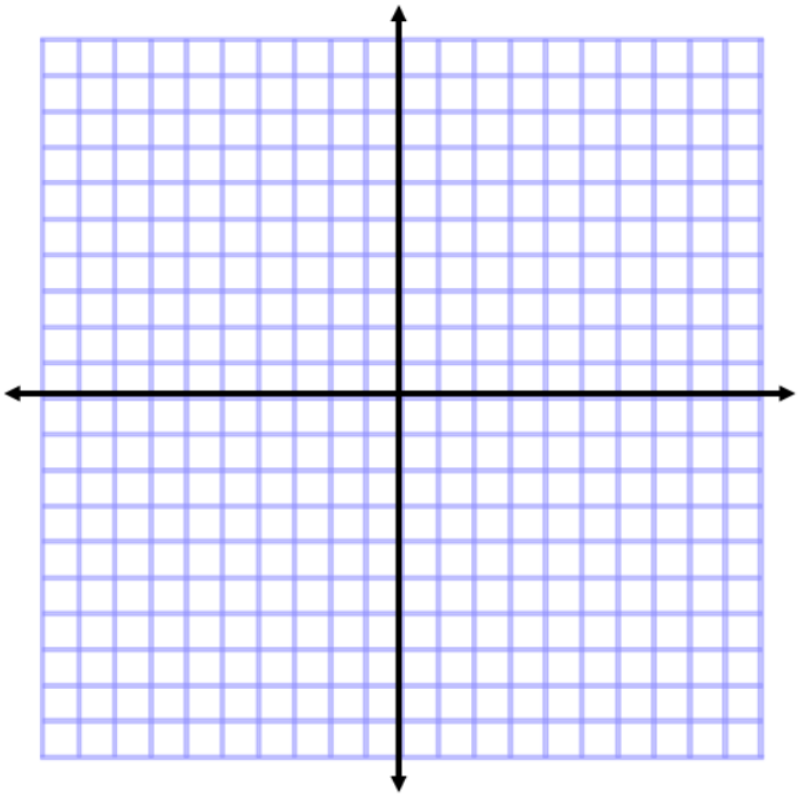
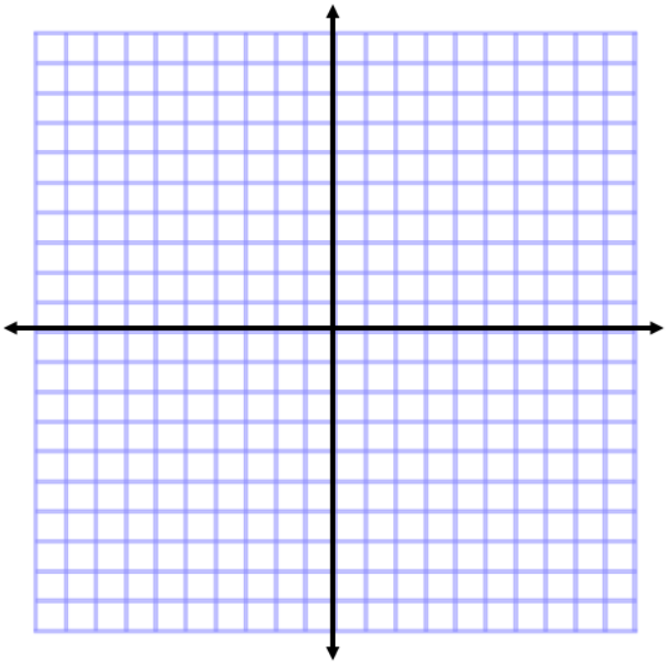


5. Two of the coordinates of a rectangle are $A(3, 7)$ and $B(3, 2)$. The rectangle has an area of 30 square units. Give the possible locations of the other two vertices by identifying their coordinates. (Use the coordinate plane to draw and check your answer.)



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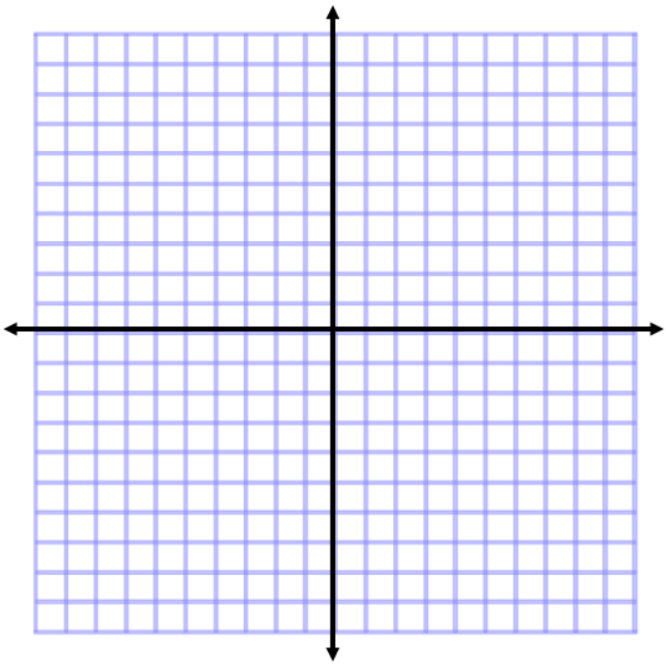
Exercises

For Exercises 1 and 2, plot the points, name the shape, and determine the area of the shape. Then write an expression that could be used to determine the area of the figure. Explain how each part of the expression corresponds to the situation.



1. $A(4, 6)$, $B(8, 6)$, $C(10, 2)$, $D(8, -3)$, $E(5, -3)$, and $F(2, 2)$

2. $X(-9, 6)$, $Y(-2, -1)$, and $Z(-8, -7)$

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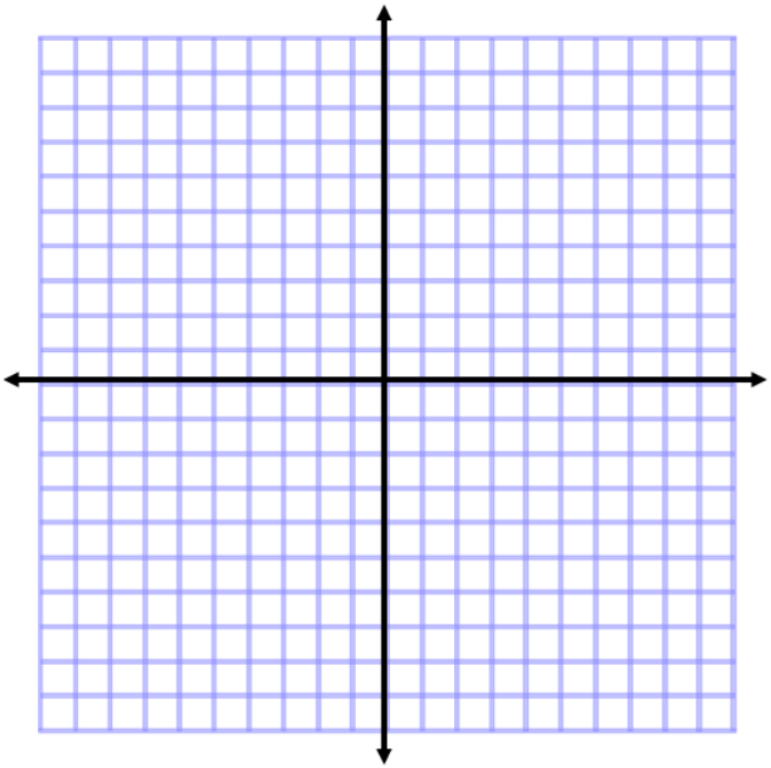
3. A rectangle with vertices located at $(-3, 4)$ and $(5, 4)$ has an area of 32 square units. Determine the location of the other two vertices.

4. Challenge: A triangle with vertices located at $(-2, -3)$ and $(3, -3)$ has an area of 20 square units. Determine one possible location of the other vertex.



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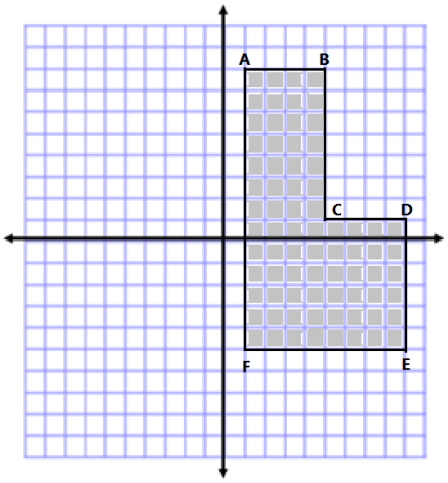
Lesson 9: Determining Area and Perimeter of Polygons on the Coordinate Plane

Classwork

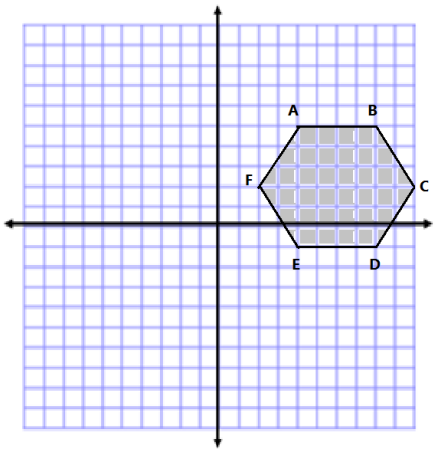
Example 1

Jasjeet has made a scale drawing of a vegetable garden she plans to make in her backyard. She needs to determine the perimeter and area to know how much fencing and dirt to purchase. Determine both the perimeter and area.



Example 2

Calculate the area of the polygon using two different methods. Write two expressions to represent the two methods, and compare the structure of the expressions.



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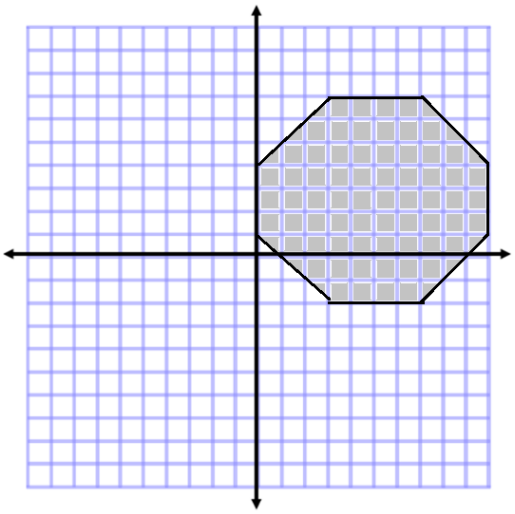
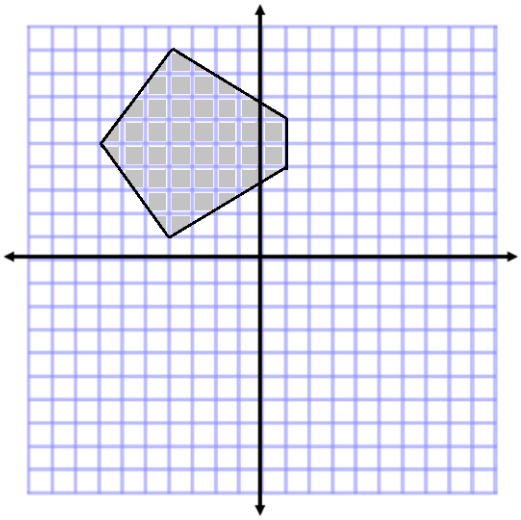
Exercises 1–2

1. Determine the area of the following shapes.

   a.



   b.



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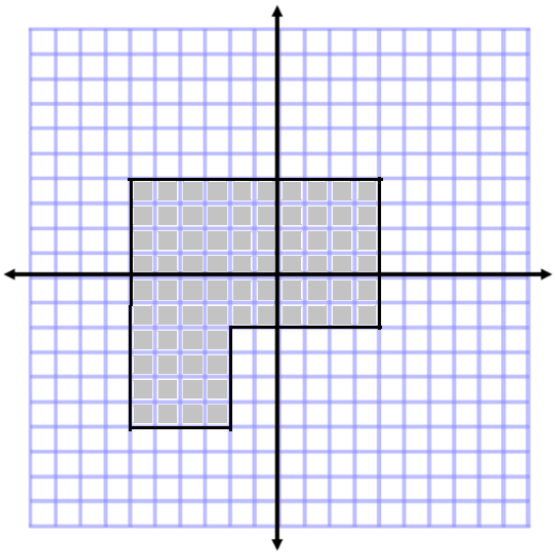
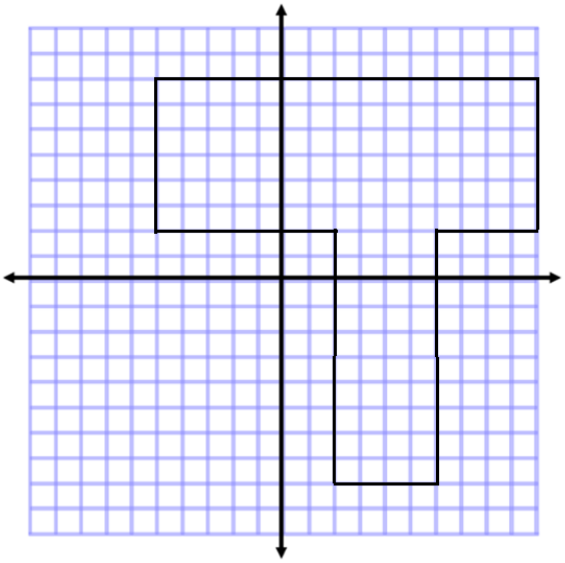
2. Determine the area and perimeter of the following shapes.

a.



b.



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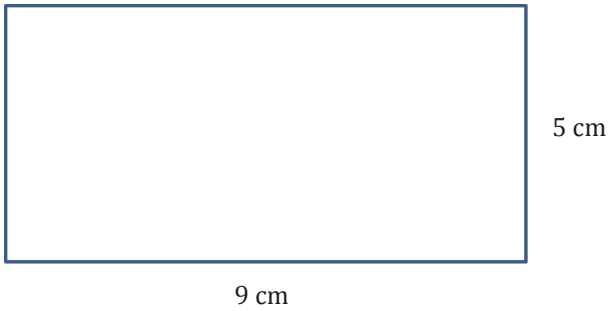
Lesson 10: Distance, Perimeter, and Area in the Real World

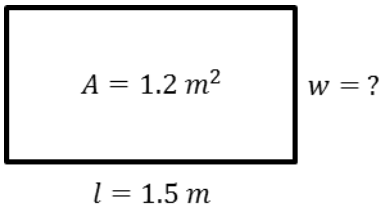
Classwork

Opening Exercises

1. Find the area and perimeter of this rectangle:



2. Find the width of this rectangle. The area is $1.2\ \mathrm{m}^2$, and the length is $1.5\ \mathrm{m}$.



Example 1: Student Desks or Tables

3. Measure the dimensions of the top of your desk.

4. How do you find the area of the top of your desk?

5. How do you find the perimeter?

6. Record these on your paper in the appropriate column.

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Exploratory Challenge

Estimate and predict the area and perimeter of each object. Then measure each object, and calculate both the area and perimeter of each.

Object or Item to be Measured	Measure-ment Units	Precision (Measure to the Nearest)	Area Prediction (Square Units)	Area (Square Units) Write the expression and evaluate it.	Perimeter Prediction (Linear Units)	Perimeter (Linear Units)
Ex: door	feet	half foot		$6\frac{1}{2}\text{ ft.} \times 3\frac{1}{2}\text{ ft.} = 22\frac{3}{4}\text{ ft}^2$		$2\left(3\frac{1}{2}\text{ ft.} + 6\frac{1}{2}\text{ ft.}\right) = 20\text{ ft.}$
Desktop						

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Optional Challenge

Object or Item to be Measured	Measurement Units	Precision (Measure to the Nearest)	Area (Square Units)	Perimeter (Linear Units)
Ex: door	feet	half foot	$6\frac{1}{2}\text{ft.} \times 3\frac{1}{2}\text{ft.} = 22\frac{3}{4}\text{ft}^2$	$2\left(3\frac{1}{2}\text{ft.} + 6\frac{1}{2}\text{ft.}\right) = 20\text{ ft.}$

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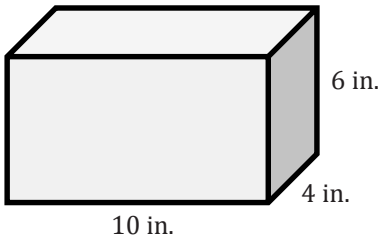
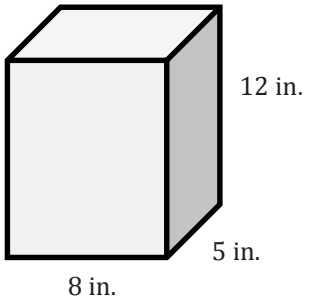
Lesson 11: Volume with Fractional Edge Lengths and Unit Cubes

Classwork

Opening Exercise

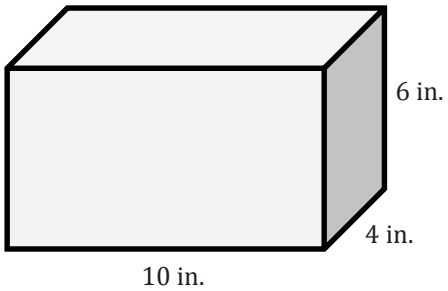
Which prism will hold more $1 \text{ in.} \times 1 \text{ in.} \times 1 \text{ in.}$ cubes? How many more cubes will the prism hold?





Example 1

A box with the same dimensions as the prism in the Opening Exercise will be used to ship miniature dice whose side lengths have been cut in half. The dice are $\frac{1}{2} \text{ in.} \times \frac{1}{2} \text{ in.} \times \frac{1}{2} \text{ in.}$ cubes. How many dice of this size can fit in the box?



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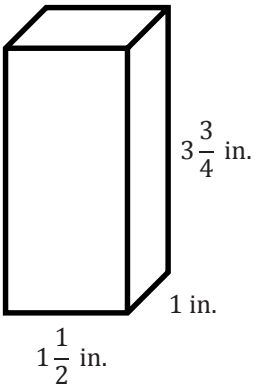
Example 2



A $\frac{1}{4}$ in. cube was used to fill the prism.

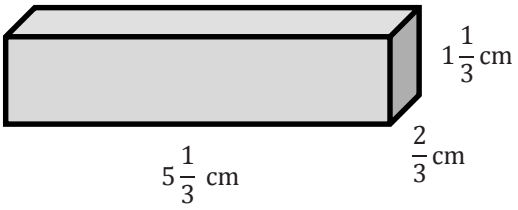
How many $\frac{1}{4}$ in. cubes will it take to fill the prism?

What is the volume of the prism?

How is the number of cubes related to the volume?

Exercises

1. Use the prism to answer the following questions.



   a. Calculate the volume.

   b. If you have to fill the prism with cubes whose side lengths are less than 1 cm, what size would be best?

   c. How many of the cubes would fit in the prism?

   d. Use the relationship between the number of cubes and the volume to prove that your volume calculation is correct.

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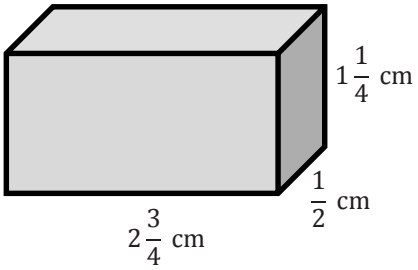
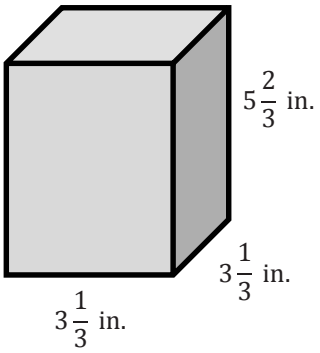
2. Calculate the volume of the following rectangular prisms.

a.



b.



3. A toy company is packaging its toys to be shipped. Some of the very small toys are placed inside a cube-shaped box with side lengths of $\frac{1}{2}$ in. These smaller boxes are then packed into a shipping box with dimensions of
$12 \text{ in.} \times 4\frac{1}{2} \text{ in.} \times 3\frac{1}{2} \text{ in.}$

a. How many small toys can be packed into the larger box for shipping?

b. Use the number of toys that can be shipped in the box to help determine the volume of the box.

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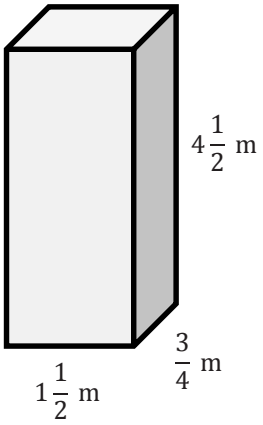
4. A rectangular prism with a volume of 8 cubic units is filled with cubes. First, it is filled with cubes with side lengths of $\frac{1}{2}$ unit. Then, it is filled with cubes with side lengths of $\frac{1}{3}$ unit.

   a. How many more of the cubes with $\frac{1}{3}$-unit side lengths than cubes with $\frac{1}{2}$-unit side lengths will be needed to fill the prism?

   b. Why does it take more cubes with $\frac{1}{3}$-unit side lengths to fill the prism?

5. Calculate the volume of the rectangular prism. Show two different methods for determining the volume.



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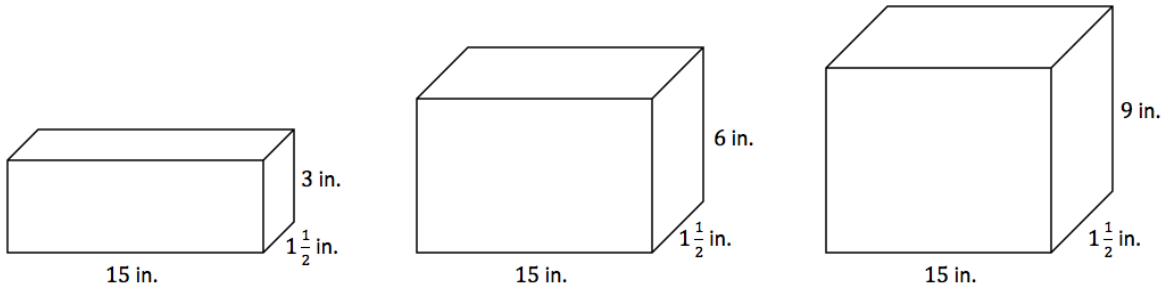
Lesson 12: From Unit Cubes to the Formulas for Volume

Classwork

Example 1



a. Write a numerical expression for the volume of each of the rectangular prisms above.

b. What do all of these expressions have in common? What do they represent?

c. Rewrite the numerical expressions to show what they have in common.

d. If we know volume for a rectangular prism as length times width times height, what is another formula for volume that we could use based on these examples?

e. What is the area of the base for all of the rectangular prisms?

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f. Determine the volume of each rectangular prism using either method.

g. How do the volumes of the first and second rectangular prisms compare? The volumes of the first and third?

Example 2

The base of a rectangular prism has an area of $3\frac{1}{4}\ \text{in}^2$. The height of the prism is $2\frac{1}{2}\ \text{in}$. Determine the volume of the rectangular prism.

Extension

A company is creating a rectangular prism that must have a volume of $6\ \text{ft}^3$. The company also knows that the area of the base must be $2\frac{1}{2}\ \text{ft}^2$. How can you use what you learned today about volume to determine the height of the rectangular prism?

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Lesson 13: The Formulas for Volume

Classwork

Example 1

Determine the volume of a cube with side lengths of $2\frac{1}{4}$ cm.

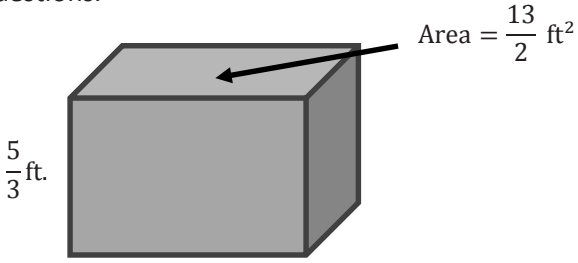
Example 2

Determine the volume of a rectangular prism with a base area of $\frac{7}{12}$ ft^2 and a height of $\frac{1}{3}$ ft.

Exercises

1. Use the rectangular prism to answer the next set of questions.

   a. Determine the volume of the prism.



   b. Determine the volume of the prism if the height of the prism is doubled.

   c. Compare the volume of the rectangular prism in part (a) with the volume of the prism in part (b). What do you notice?

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d. Complete and use the table below to determine the relationship between the height and volume.

Height in Feet	Volume in Cubic Feet
$\frac{5}{3}$	$\frac{65}{6}$
$\frac{10}{3}$	$\frac{130}{6}$
$\frac{15}{3}$	
$\frac{20}{3}$	

What happened to the volume when the height was tripled?

What happened to the volume when the height was quadrupled?

What conclusions can you make when the base area stays constant and only the height changes?

2.

a. If B represents the area of the base and h represents the height, write an expression that represents the volume.

b. If we double the height, write an expression for the new height.

c. Write an expression that represents the volume with the doubled height.

d. Write an equivalent expression using the commutative and associative properties to show the volume is twice the original volume.

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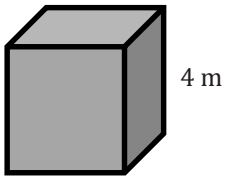
3. Use the cube to answer the following questions.

   a. Determine the volume of the cube.



   b. Determine the volume of a cube whose side lengths are half as long as the side lengths of the original cube.

   c. Determine the volume if the side lengths are one-fourth as long as the original cube's side lengths.

   d. Determine the volume if the side lengths are one-sixth as long as the original cube's side length.

   e. Explain the relationship between the side lengths and the volumes of the cubes.

4. Check to see if the relationship you found in Exercise 3 is the same for rectangular prisms.

   a. Determine the volume of the rectangular prism.

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b. Determine the volume if all of the sides are half as long as the original lengths.

c. Determine the volume if all of the sides are one-third as long as the original lengths.

d. Is the relationship between the side lengths and the volume the same as the one that occurred in Exercise 3? Explain your answer.

5.

a. If e represents an edge length of the cube, create an expression that shows the volume of the cube.

b. If we divide the edge lengths by three, create an expression for the new edge length.

c. Write an expression that represents the volume of the cube with one-third the side length.

d. Write an equivalent expression to show that the volume is $\frac{1}{27}$ of the original volume.

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Lesson 14: Volume in the Real World

Classwork

Example 1

a. The area of the base of a box is $16\ \text{in}^2$. The volume of the box is $24\ \text{in}^3$. Determine the height of the box.

b. The area of the base of a sandbox is $9\frac{1}{2}\ \text{ft}^2$. The volume of the sandbox is $7\frac{1}{8}\ \text{ft}^3$. Determine the height of the sandbox.

c. The sandbox was filled with sand, but after kids played, some of the sand spilled out. Now, the sand is at a height of $\frac{1}{2}$ ft. Determine the volume of the sand.

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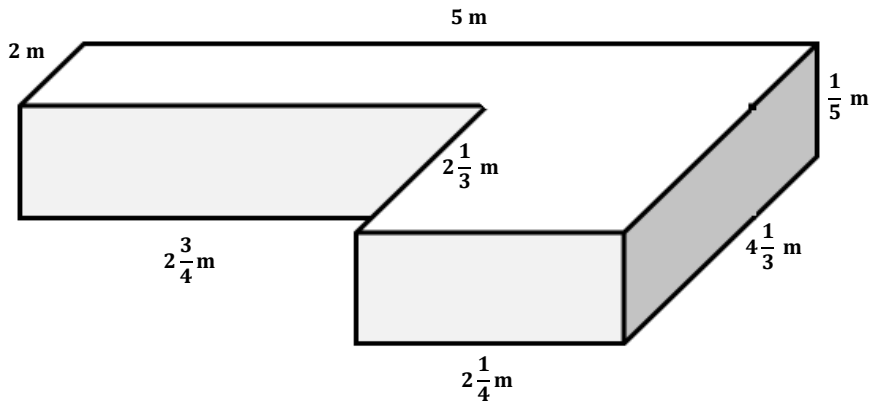
Example 2

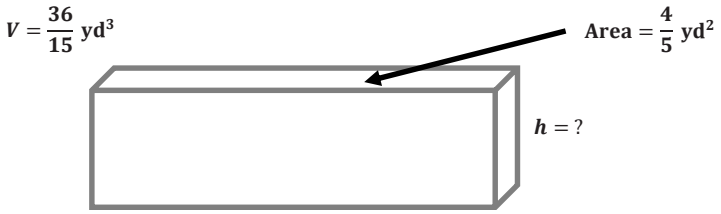
A special order sandbox has been created for children to use as an archeological digging area at the zoo. Determine the volume of the sandbox.



Exercises

1.

a. The volume of the rectangular prism is given. Determine the missing measurement using a one-step equation.



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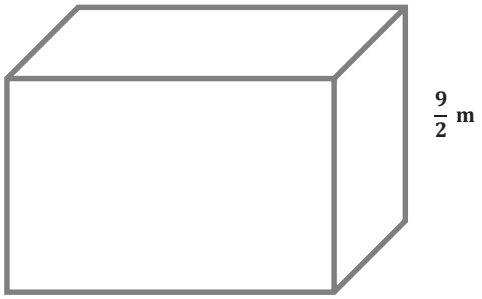
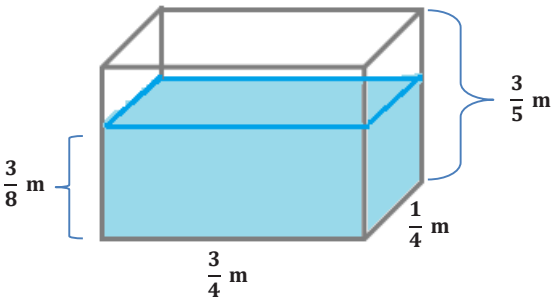
b. The volume of the box is $\frac{45}{6}$ m^3. Determine the area of the base using a one-step equation.



2. Marissa's fish tank needs to be filled with more water.

a. Determine how much water the tank can hold.



b. Determine how much water is already in the tank.

c. How much more water is needed to fill the tank?

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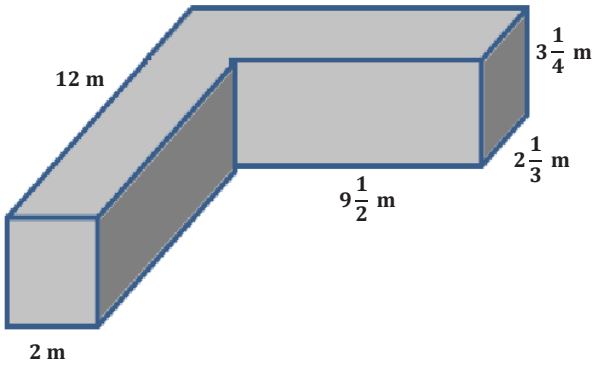
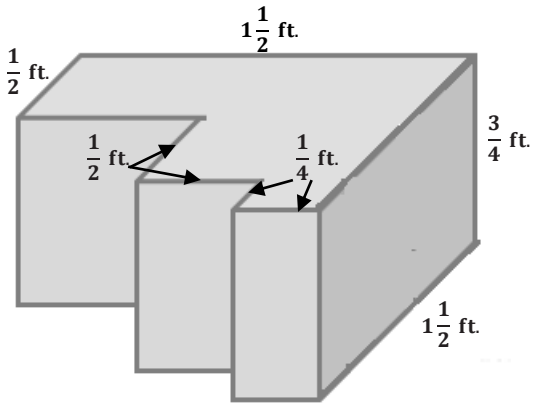
3. Determine the volume of the composite figures.

a.



b.



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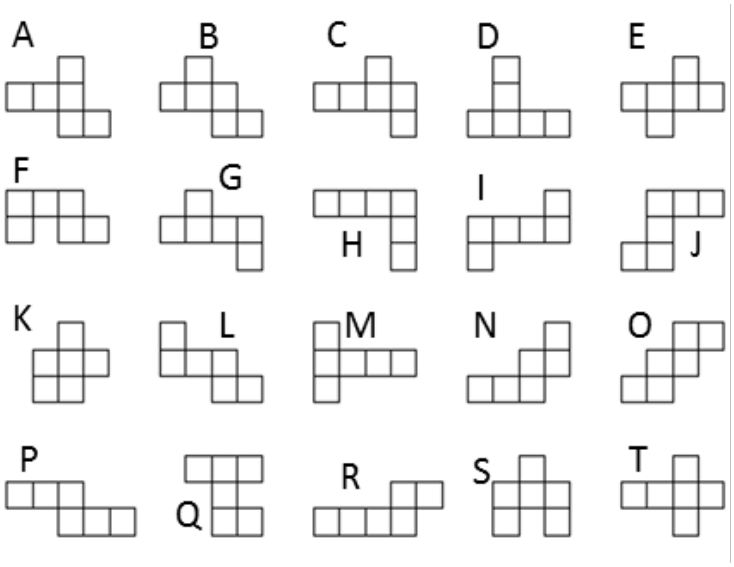
Lesson 15: Representing Three-Dimensional Figures Using Nets

Classwork

Exercise: Cube

Nets are two-dimensional figures that can be folded up into three-dimensional solids. Some of the drawings below are nets of a cube. Others are not cube nets; they can be folded, but not into a cube.



c. Experiment with the larger cut out patterns provided. Shade in each of the figures above that will fold into a cube.

d. Write the letters of the figures that can be folded up into a cube.

e. Write the letters of the figures that cannot be folded up into a cube.

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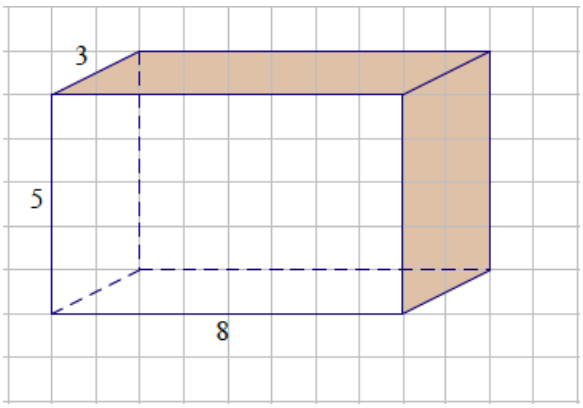
Lesson 16: Constructing Nets

Classwork

Opening Exercise

Sketch all six faces of the rectangular prism in the area below. Label the dimensions.



Notes from discussion:

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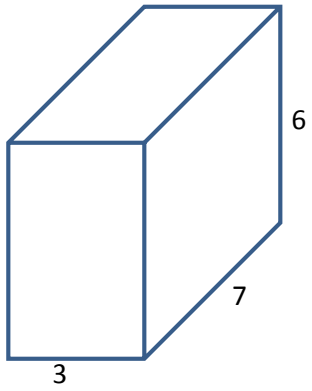
Exploratory Challenge 1: Rectangular Prisms

a. Use the measurements from the solid figures to cut and arrange the faces into a net.



b. A juice box measures 4 inches high, 3 inches long, and 2 inches wide. Cut and arrange all 6 faces into a net.

c. Challenge: Write a numerical expression for the total area of the net for part (b). Explain each term in your expression.

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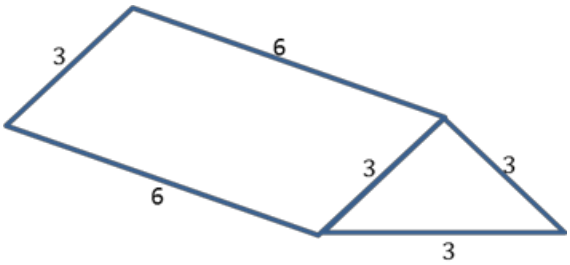
Exploratory Challenge 2: Triangular Prism

Use the measurements from the triangular prism to cut and arrange the faces into a net.



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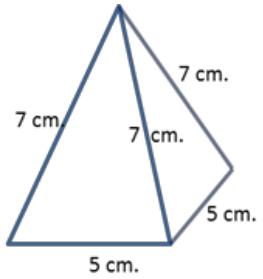
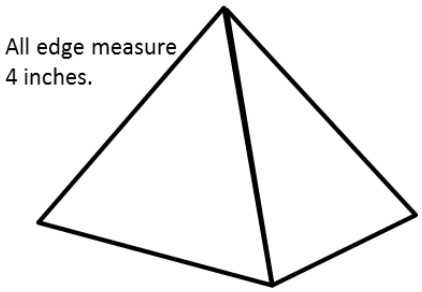
Exploratory Challenge 3: Pyramids

Pyramids are named for the shape of the base.

a. Use the measurements from this square pyramid to cut and arrange the faces into a net. Test your net to be sure it folds into a square pyramid.



b. A triangular pyramid that has equilateral triangles for faces is called a tetrahedron. Use the measurements from this tetrahedron to cut and arrange the faces into a net.



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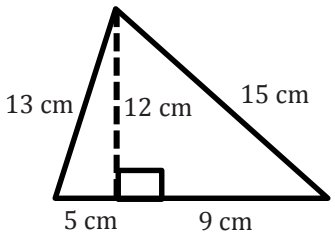
Lesson 17: From Nets to Surface Area

Classwork

Opening Exercise

a. Write a numerical equation for the area of the figure below. Explain and identify different parts of the figure.

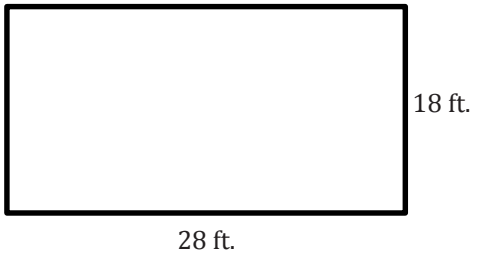
i.



ii. How would you write an equation that shows the area of a triangle with base b and height h?

b. Write a numerical equation for the area of the figure below. Explain and identify different parts of the figure.

i.



ii. How would you write an equation that shows the area of a rectangle with base b and height h?

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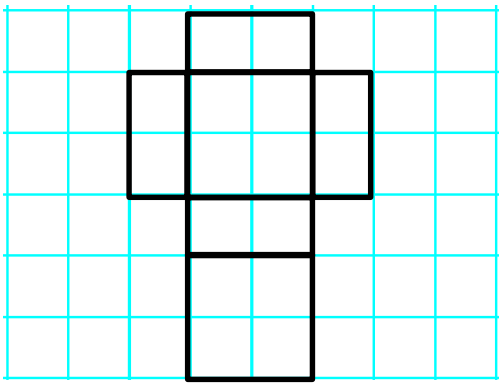
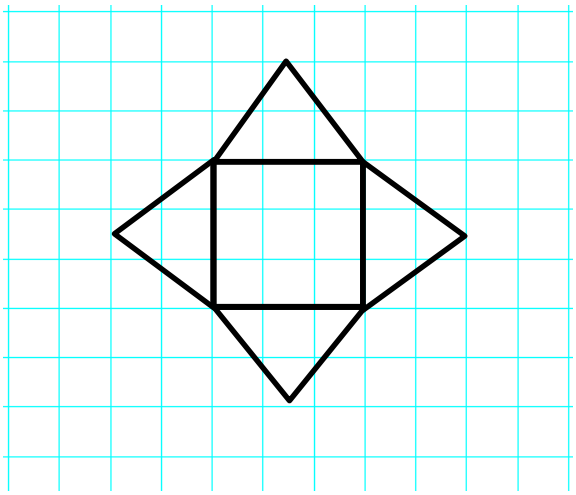
Example 1

Use the net to calculate the surface area of the figure.



Example 2

Use the net to write an expression for surface area.



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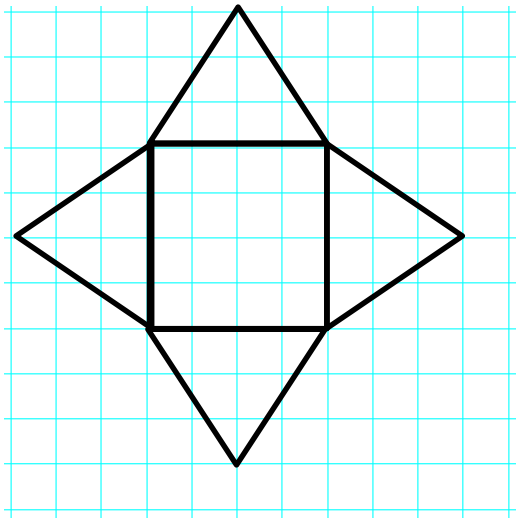
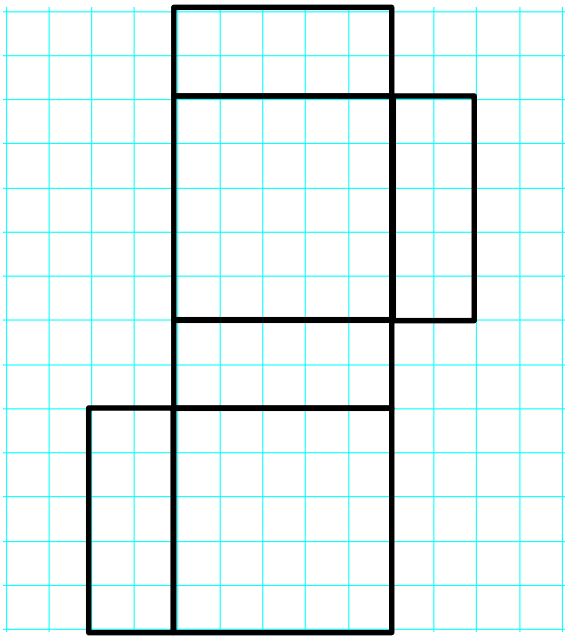
Exercises

Name the solid the net would create, and then write an expression for the surface area. Use the expression to determine the surface area. Assume that each box on the grid paper represents a $1\text{ cm} \times 1\text{ cm}$ square. Explain how the expression represents the figure.

1.



2.



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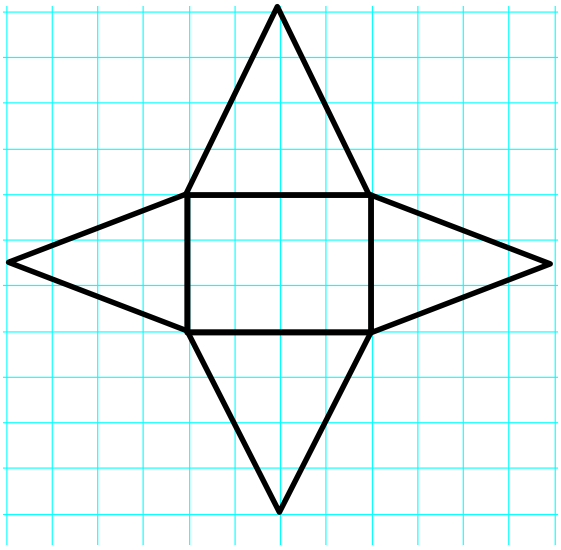
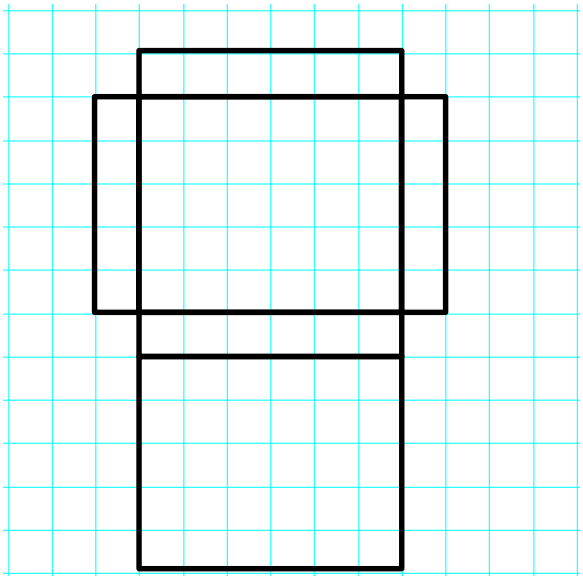
3.



4.



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Lesson 18: Determining Surface Area of Three-Dimensional Figures

Classwork

Opening Exercise

a. What three-dimensional figure will the net create?

b. Measure (in inches) and label each side of the figure.

c. Calculate the area of each face, and record this value inside the corresponding rectangle.

d. How did we compute the surface area of solid figures in previous lessons?

e. Write an expression to show how we can calculate the surface area of the figure above.

f. What does each part of the expression represent?

g. What is the surface area of the figure?

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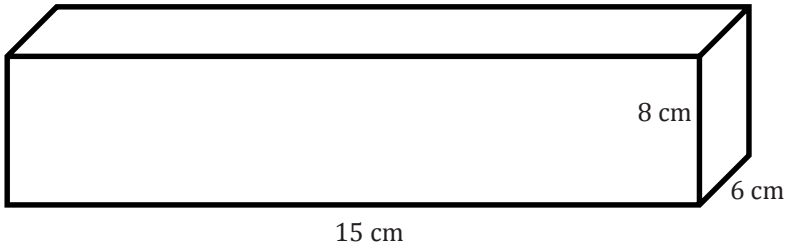
Example 1

Fold the net used in the Opening Exercise to make a rectangular prism. Have the two faces with the largest area be the bases of the prism. Fill in the second row of the table below.

Area of Top (base)	Area of Bottom (base)	Area of Front	Area of Back	Area of Left Side	Area of Right Side

Examine the rectangular prism below. Complete the table.



Area of Top (base)	Area of Bottom (base)	Area of Front	Area of Back	Area of Left Side	Area of Right Side

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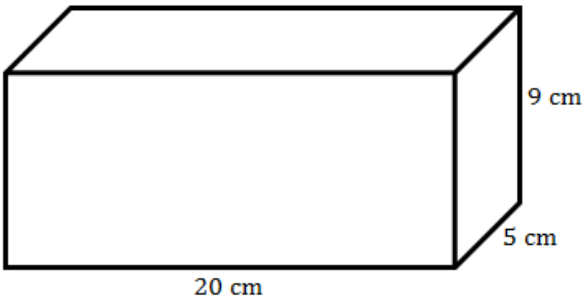
Example 2

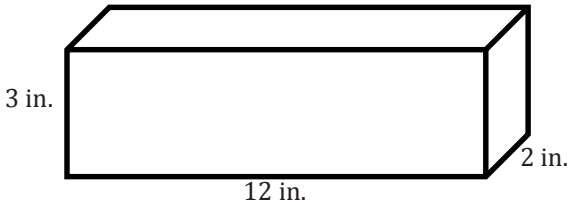
Calculate the surface area.



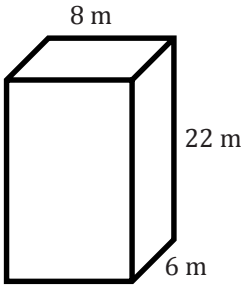
Exercises 1–3

1. Calculate the surface area of each of the rectangular prisms below.

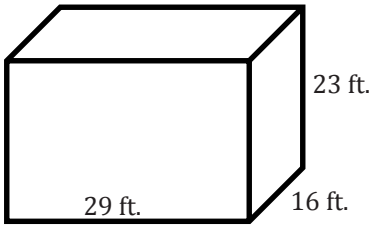
a.



b.



c.



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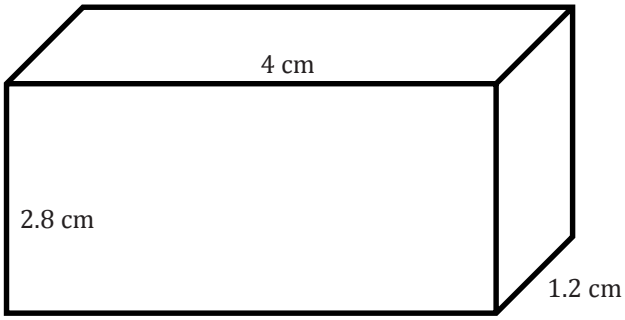
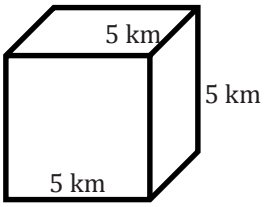
d.



2. Calculate the surface area of the cube.



3. All the edges of a cube have the same length. Tony claims that the formula $SA = 6s^2$, where s is the length of each side of the cube, can be used to calculate the surface area of a cube.

   a. Use the dimensions from the cube in Problem 2 to determine if Tony's formula is correct.

   b. Why does this formula work for cubes?

   c. Becca does not want to try to remember two formulas for surface area, so she is only going to remember the formula for a cube. Is this a good idea? Why or why not?

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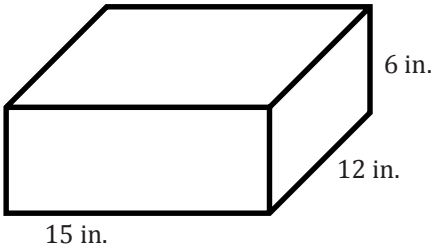
Lesson 19: Surface Area and Volume in the Real World

Classwork

Opening Exercise

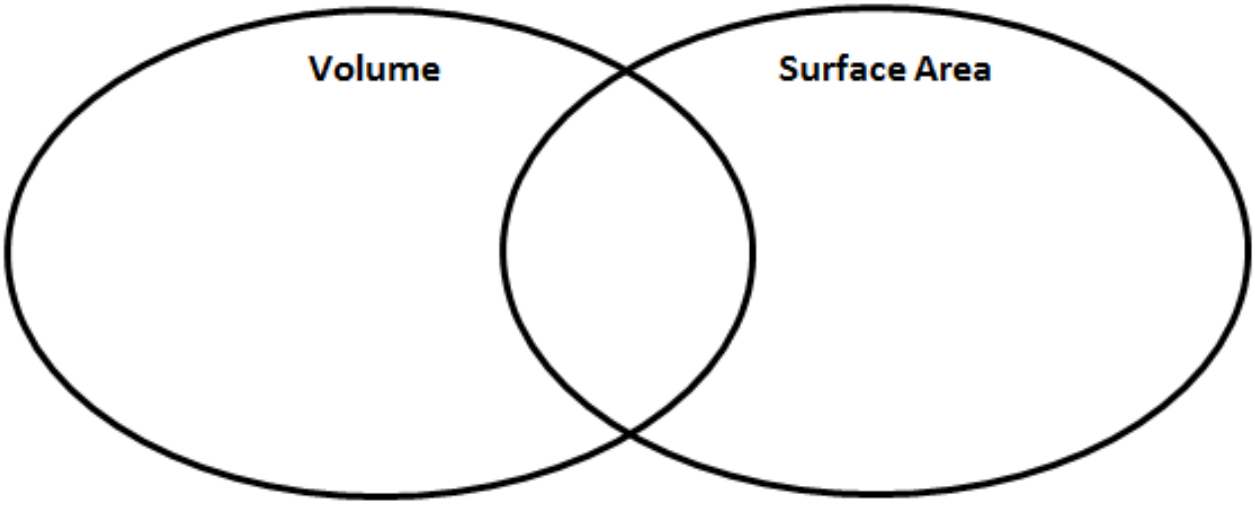
A box needs to be painted. How many square inches will need to be painted to cover every surface?



A juice box is 4 in. tall, 1 in. wide, and 2 in. long. How much juice fits inside the juice box?

How did you decide how to solve each problem?

Discussion



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Example 1

Vincent put logs in the shape of a rectangular prism. He built this rectangular prism of logs outside his house. However, it is supposed to snow, and Vincent wants to buy a cover so the logs will stay dry. If the pile of logs creates a rectangular prism with these measurements:

$$33 \text{ cm long, } 12 \text{ cm wide, and } 48 \text{ cm high,}$$

What is the minimum amount of material needed to make a cover for the wood pile?

Exercises 1–6

Use your knowledge of volume and surface area to answer each problem.

1. Quincy Place wants to add a pool to the neighborhood. When determining the budget, Quincy Place determined that it would also be able to install a baby pool that required less than 15 cubic feet of water. Quincy Place has three different models of a baby pool to choose from.

   Choice One: $5 \text{ feet} \times 5 \text{ feet} \times 1 \text{ foot}$

   Choice Two: $4 \text{ feet} \times 3 \text{ feet} \times 1 \text{ foot}$

   Choice Three: $4 \text{ feet} \times 2 \text{ feet} \times 2 \text{ feet}$

   Which of these choices is best for the baby pool? Why are the others not good choices?

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2. A packaging firm has been hired to create a box for baby blocks. The firm was hired because it could save money by creating a box using the least amount of material. The packaging firm knows that the volume of the box must be $18\ \text{cm}^3$.

   a. What are possible dimensions for the box if the volume must be exactly $18\ \text{cm}^3$?

   b. Which set of dimensions should the packaging firm choose in order to use the least amount of material? Explain.

3. A gift has the dimensions of $50\ \text{cm} \times 35\ \text{cm} \times 5\ \text{cm}$. You have wrapping paper with dimensions of $75\ \text{cm} \times 60\ \text{cm}$. Do you have enough wrapping paper to wrap the gift? Why or why not?

4. Tony bought a flat rate box from the post office to send a gift to his mother for Mother's Day. The dimensions of the medium size box are $14\ \text{inches} \times 12\ \text{inches} \times 3.5\ \text{inches}$. What is the volume of the largest gift he can send to his mother?

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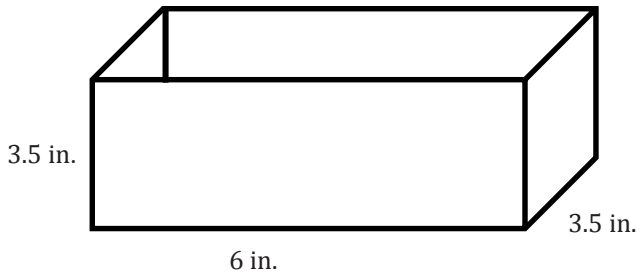
5. A cereal company wants to change the shape of its cereal box in order to attract the attention of shoppers. The original cereal box has dimensions of $8 \text{ inches} \times 3 \text{ inches} \times 11 \text{ inches}$. The new box the cereal company is thinking of would have dimensions of $10 \text{ inches} \times 10 \text{ inches} \times 3 \text{ inches}$.

   a. Which box holds more cereal?

   b. Which box requires more material to make?

6. Cinema theaters created a new popcorn box in the shape of a rectangular prism. The new popcorn box has a length of 6 inches, a width of 3.5 inches, and a height of 3.5 inches but does not include a lid.



   a. How much material is needed to create the box?

   b. How much popcorn does the box hold?

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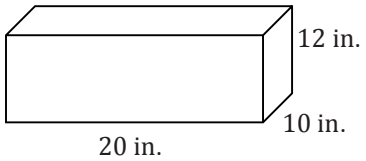
Lesson 19a (Optional): Applying Surface Area and Volume to Aquariums

Classwork

Opening Exercise

Determine the volume of this aquarium.



Mathematical Modeling Exercise: Using Ratios and Unit Rate to Determine Volume

For his environmental science project, Jamie is creating habitats for various wildlife including fish, aquatic turtles, and aquatic frogs. For each of these habitats, he will use a standard aquarium with length, width, and height dimensions measured in inches, identical to the aquarium mentioned in the Opening Exercise. To begin his project, Jamie will need to determine the volume, or cubic inches, of water that will fill the aquarium.

Use the table below to determine the unit rate of gallons/cubic inches.

Gallons	Cubic Inches
1	
2	462
3	693
4	924
5	1,155

Determine the volume of the aquarium.

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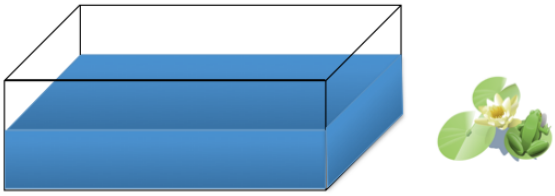
Exercise 1



a. Determine the volume of the tank when filled with 7 gallons of water.

b. Work with your group to determine the height of the water when Jamie places 7 gallons of water in the aquarium.

Exercise 2

a. Use the table from Example 1 to determine the volume of the aquarium when Jamie pours 3 gallons of water into the tank.

b. Use the volume formula to determine the missing height dimension.

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Exercise 3

a. Using the table of values below, determine the unit rate of liters to gallon.

Gallons	Liters
1	
2	7.57
4	15.14

b. Using this conversion, determine the number of liters you will need to fill the 10-gallon tank.

c. The ratio of the number of centimeters to the number of inches is $2.54 : 1$. What is the unit rate?

d. Using this information, complete the table to convert the heights of the water in inches to the heights of the water in centimeters Jamie will need for his project at home.

Height in Inches	Convert to Centimeters	Height in Centimeters
1	$2.54 \frac{\text{centimeters}}{\text{inch}} \times 1 \text{ inch}$	2.54
3.465		
8.085		
11.55		

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Exercise 4

a. Determine the amount of plastic film the manufacturer uses to cover the aquarium faces. Draw a sketch of the aquarium to assist in your calculations. Remember that the actual height of the aquarium is 12 inches.

b. We do not include the measurement of the top of the aquarium since it is open without glass and does not need to be covered with film. Determine the area of the top of the aquarium, and find the amount of film the manufacturer will use to cover only the sides, front, back, and bottom.

c. Since Jamie will need three aquariums, determine the total surface area of the three aquariums.

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